Boyer's Covariant Ether, the Rindler Horizon and the Genesis of Inertia and Geometry

Complete classical derivations and speculative directions

Working notes

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Abstract

We study Boyer's classical zero-point field (stochastic electrodynamics), whose ω^3 spectrum is the unique Lorentz-invariant spectrum of random radiation. We derive in full what this field becomes along a uniformly accelerated worldline: the sampled correlation function becomes stationary and its spectrum takes the form $\frac{\omega}{4\pi}\coth(\pi\omega c/a)$, i.e. exactly "zero point + Planck" at the Unruh temperature, without any quantum. We then analyse what, in this classical reading, replaces quantum entanglement across the Rindler horizon: neither a filtering of wavelengths nor a disappearance of correlations, but a *marginalization* of a globally correlated Gaussian field, with a trans-horizon correlation coefficient $\gamma(\omega) = 1/\cosh(\pi\omega c/a)$ which we compute. We put this result to work in a re-reading of McCulloch's Quantized Inertia, showing that the scaling law $\Delta m/m \sim -c^2/(a\Theta)$ is robust against the details of the infrared filtering. The cosmological horizon is then integrated as a second horizon: the sampling theorem generalizes through the substitution $a \to \kappa = \sqrt{a^2 + c^2H^2}$, and the disappearance of inertia at low acceleration receives a correlational explanation — the front–rear asymmetry of the unshared energy vanishes when the Rindler horizon reaches the Hubble horizon — with a computed coefficient ($24/\pi^2 \approx 2.4$) close to McCulloch's. We finally propose an extension of the covariant ether to General Relativity (Hadamard condition, stochastic gravity, Deser–Levin temperature floor) and show that the Minkowski metric is fully *reconstructed* from Boyer's correlations, before examining under what conditions a field could *engender* a geometry rather than merely respect one (self-consistent fixed point, Sakharov's induced gravity, Jacobson's equation of state). Established results, reviews and original speculations are explicitly distinguished.

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1 Introduction, conventions and epistemic status

This document addresses four chained questions. *(i)* What becomes of Boyer's classical zero-point field, with its Lorentz-invariant ω^3 spectrum, for a uniformly accelerated observer? *(ii)* In this classical version of the Unruh effect, what replaces the quantum entanglement between the two sides of the Rindler horizon: a filtering of the wavelengths that would overflow the horizon, or a suppression of correlations — and in what form? *(iii)* How should this "covariant ether" be extended to General Relativity? *(iv)* Can the Minkowski metric be recovered from Boyer's radiation and, more ambitiously, what correlations would a field need in order to *engender* a geometry?

Question *(ii)* is the pivot: it is what gives purchase to McCulloch's theory of Quantized Inertia (MiHsC), in which inertia arises from the depletion of Unruh radiation by the action of a horizon. We shall therefore treat it with particular care.

Epistemic status. Every section carries one of the following three labels. **[Established]**: proven result, complete derivation supplied. **[Review]**: faithful synthesis of an existing literature, including non-consensual work (MiHsC), treated as a working hypothesis and not as a target for refutation. **[Speculation]**: original proposal of this document, owned as such. Known objections are mentioned as *structuring constraints* for the speculation, never as verdicts.

Conventions. Signature $(-,+,+,+)$, $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$. We write $\Delta x = x - x'$. Derivations are carried out in natural units $\hbar = c = 1$ (and $k_B = 1$), with physical results restored in SI units. The statistical average over the random ensemble is denoted $\langle\cdot\rangle$. To lighten the notation, most derivations are done for a massless scalar field φ; the electromagnetic case is treated in parallel wherever it differs.

2 Boyer's classical zero-point field [Established]

2.1 Definition of the random field

Stochastic electrodynamics (Marshall [1], Boyer [2, 3]) postulates that at zero temperature space is not empty of field: it is filled with a classical *random*

radiation, homogeneous, isotropic and stationary, with uncorrelated phases. For the model scalar field:

$$\varphi(t,\mathbf{r}) = \sqrt{\hbar}\int \frac{\mathrm{d}^3k}{\sqrt{(2\pi)^3\,2\omega_k}}\left[a(\mathbf{k})\,e^{\,i(\mathbf{k}\cdot\mathbf{r}-\omega_k t)} + \text{c.c.}\right], \qquad \omega_k = c\,|\mathbf{k}|, \quad (1)$$

where the complex amplitudes $a(\mathbf{k}) = |a(\mathbf{k})|\,e^{\,i\theta(\mathbf{k})}$ have independent phases $\theta(\mathbf{k})$, uniformly distributed on $[0, 2\pi)$, with the second moments

$$\big\langle a(\mathbf{k})\,a^*(\mathbf{k}')\big\rangle = \tfrac{1}{2}\,\delta^3(\mathbf{k}-\mathbf{k}'), \qquad \big\langle a(\mathbf{k})\,a(\mathbf{k}')\big\rangle = 0. \quad (2)$$

For any smeared observable, the central limit theorem renders the field Gaussian: it is then *entirely* characterized by its two-point correlation function. From (1)–(2):

$$C(x,x') \equiv \big\langle \varphi(x)\,\varphi(x')\big\rangle = \hbar\,\mathrm{Re}\int \frac{\mathrm{d}^3k}{(2\pi)^3\,2\omega_k}\,e^{\,i(\mathbf{k}\cdot\Delta\mathbf{r}-\omega_k\Delta t)}. \quad (3)$$

The energy density follows in the same way:

$$u = \tfrac{1}{2}\big\langle \dot\varphi^2 + c^2(\nabla\varphi)^2\big\rangle = \int \frac{\mathrm{d}^3k}{(2\pi)^3}\,\frac{\hbar\omega_k}{2} = \int_0^\infty \rho(\omega)\,\mathrm{d}\omega, \qquad \rho_{\text{scal}}(\omega) = \frac{\hbar\,\omega^3}{4\pi^2c^3}. \quad (4)$$

Each mode thus carries the energy $\hbar\omega/2$. For the electromagnetic field (two polarizations), Boyer's spectrum is

$$\boxed{\rho(\omega) = \frac{\hbar\,\omega^3}{2\pi^2c^3}} \quad (5)$$

Let us insist: the theory is *classical*. The constant $\hbar$ plays no quantization role whatsoever; it merely sets the *scale* of the random field, as a new constant of nature ("$\hbar$ without quanta").

2.2 Lorentz invariance and uniqueness of the ω^3 spectrum

Proposition 1 (Boyer 1969) *The spectrum (5) is, up to a global scale factor, the unique spectrum of isotropic random radiation whose correlation functions are Lorentz invariant. In particular, such radiation defines* no preferred frame*: it is an "ether" compatible with Special Relativity.*

Derivation. Rewrite (3) as an integral over the future light cone $\mathcal{C}_+ = \{k^\mu : k^2 = 0,\ k^0 > 0\}$ (units $\hbar = c = 1$):

$$C(x,x') = \mathrm{Re}\int_{\mathcal{C}_+} \mathrm{d}\mu(k)\;g(\omega)\;e^{\,ik_\mu\Delta x^\mu}, \qquad \mathrm{d}\mu(k) = \frac{\mathrm{d}^3k}{(2\pi)^3\,2\omega}, \quad (6)$$

where a general spectral weight $g(\omega)$ has been allowed (Boyer's field corresponds to $g \equiv 1$). Two elementary facts: (a) the measure $\mathrm{d}^3k/2\omega$ is the

Lorentz-invariant measure on $\mathcal{C}_+$; (b) the phase $k_\mu \Delta x^\mu$ is an invariant. Consequently C is an invariant biscalar *if and only if* the weight g is an invariant function on the cone. Now the future light cone (with the origin removed) is a *single orbit* of the proper orthochronous Lorentz group: every invariant function on it is constant. Hence $g(\omega) = \text{const}$, i.e. energy $\propto \omega$ per mode, i.e. $\rho(\omega) \propto \omega^3$ by (4). □

Physical reading (Doppler). Under a boost, every mode undergoes Doppler shift and aberration, $\omega' = \gamma\omega(1 - \beta\cos\theta)$; the specific intensity I_ω of a radiation field transforms in such a way that I_ω/ω^3 is invariant (Liouville's theorem on the phase space of rays). A spectrum that has *the same functional form* in all frames must therefore satisfy $I_\omega \propto \omega^3$: the forward Doppler hardening and the backward reddening compensate exactly, mode by mode. This is the precise sense in which Boyer's ether is undetectable by any Michelson–Morley or drag-type experiment: it has no rest frame. It realizes the "relativistic" ether contemplated by Dirac [6].

2.3 Dictionary with the quantum vacuum: the symmetric part

The comparison with quantum field theory is structurally simple and worth stating once and for all. The function (3) coincides *exactly* with the symmetric part (Hadamard function) of the Minkowski vacuum:

$$C(x, x') = \tfrac{1}{2} \left\langle 0 \right| \{\hat{\varphi}(x), \hat{\varphi}(x')\} \left| 0 \right\rangle. \tag{7}$$

Stochastic electrodynamics is thus the theory that takes seriously, as a genuine classical probability distribution, the (Gaussian, positive) Wigner function of the quantum vacuum. Every prediction that depends only on *symmetrized* correlation functions is identical in the two theories: the Casimir effect, van der Waals forces, the size of the oscillator ground state, diamagnetism, and — as we are about to see — the Unruh effect. What the classical field lacks is the *antisymmetric* part (the commutator), which carries non-commutativity: photon statistics, antibunching, violations of Bell inequalities. This point will reappear decisively in section 4.

3 Uniform acceleration: complete derivation of the classical thermal spectrum [Established]

3.1 The Rindler trajectory and the interval

Consider the observer with constant proper acceleration a (hyperbolic motion):

$$c\,t(\tau) = \frac{c^2}{a}\sinh\frac{a\tau}{c}, \qquad x(\tau) = \frac{c^2}{a}\cosh\frac{a\tau}{c}, \qquad y = z = 0, \tag{8}$$

where τ is the proper time. With $\Delta\tau = \tau - \tau'$ and the hyperbolic addition identities,

$$c\,\Delta t = \frac{2c^2}{a}\,\cosh\frac{a(\tau+\tau')}{2c}\,\sinh\frac{a\,\Delta\tau}{2c}, \quad \Delta x = \frac{2c^2}{a}\,\sinh\frac{a(\tau+\tau')}{2c}\,\sinh\frac{a\,\Delta\tau}{2c}, \tag{9}$$

whence the interval along the trajectory:

$$\Delta x^2 - c^2\Delta t^2 = -\frac{4c^4}{a^2}\,\sinh^2\frac{a\,\Delta\tau}{2c}. \tag{10}$$

The crucial fact: the interval depends only on $\Delta\tau$. The sampling of the field along (8) is therefore a random process that is *stationary* in proper time, which permits a spectral analysis.

3.2 Sampled correlation and power spectrum

In units $\hbar = c = 1$, the integral (3) evaluates in closed form (massless scalar field):

$$C(x,x') = \mathrm{Re}\,W(x,x'), \qquad W(x,x') = \frac{1}{4\pi^2}\,\frac{1}{|\Delta\mathbf{r}|^2 - (\Delta t - i\varepsilon)^2}, \tag{11}$$

the $i\varepsilon$ prescription defining the distribution (the real part corresponds, for the classical field, to the principal value). Inserting (10):

$$W(\Delta\tau) = -\frac{a^2}{16\pi^2}\,\frac{1}{\sinh^2\!\left[\frac{a}{2}(\Delta\tau - i\varepsilon)\right]}. \tag{12}$$

In the inertial limit $a \to 0$ one recovers $W = -1/[4\pi^2(\Delta\tau - i\varepsilon)^2]$: the zero point sampled by an inertial observer.

Lemma 1 (Fourier transform of $\sinh^{-2}$) *Using the partial-fraction expansion*

$$\frac{1}{\sinh^2 z} = \sum_{n\in\mathbb{Z}} \frac{1}{(z - i\pi n)^2}, \tag{13}$$

equation (12) can be rewritten as

$$W(\Delta\tau) = -\frac{1}{4\pi^2}\sum_{n\in\mathbb{Z}} \frac{1}{\left(\Delta\tau - i\varepsilon - 2\pi i n/a\right)^2}, \tag{14}$$

and the contour integral (closing in the lower half-plane for $\omega > 0$, double poles at $\Delta\tau = 2\pi i n/a$, $n \le -1$) yields

$$F(\omega) \equiv \int_{-\infty}^{+\infty} \mathrm{d}\Delta\tau\; e^{-i\omega\Delta\tau}\, W(\Delta\tau) = \frac{\omega}{2\pi}\,\frac{1}{e^{2\pi\omega/a} - 1} \qquad (\omega > 0). \tag{15}$$

The function F satisfies the detailed-balance relation (KMS condition)

$$F(-\omega) = e^{\,2\pi\omega/a}\, F(\omega), \tag{16}$$

the signature of a thermal equilibrium state at the temperature $T = a/2\pi$, i.e. in SI units the **Unruh temperature**

$$\boxed{T_U = \frac{\hbar\, a}{2\pi c\, k_B} \;\approx\; 4.05 \times 10^{-21} \left(\frac{a}{1\ \mathrm{m\,s^{-2}}}\right) \mathrm{K}.} \tag{17}$$

The *classical* field samples only the symmetric part $C = \mathrm{Re}\, W$, whose power spectrum is

$$S_a(\omega) = \int \mathrm{d}\Delta\tau\; e^{-i\omega\Delta\tau}\, C(\Delta\tau) = \tfrac{1}{2}\big[F(\omega)+F(-\omega)\big] = \frac{1}{2\pi}\,\omega\left[\frac{1}{2} + \frac{1}{e^{2\pi\omega/a}-1}\right], \tag{18}$$

that is, in compact form,

$$\boxed{S_a(\omega) = \frac{\omega}{4\pi}\,\coth\!\left(\frac{\pi\omega}{a}\right) \;\xrightarrow[a\to 0]{}\; \frac{\omega}{4\pi}.} \tag{19}$$

3.3 Physical reading: thermality as a sampling effect

The result (19) is Boyer's central theorem [4, 5]: *the classical zero-point field, sampled along a hyperbola, possesses exactly the "zero point + Planck" statistics at the temperature* T_U. There is no particle creation, no quanta, no change of state of the field: it is *the same* field, the same realization of the randomness, but read along a worldline whose time-dependent Doppler modulation converts the ω^3 spectrum into a thermal one. A classical dipole oscillator riding the trajectory reaches equilibrium with this sampled spectrum and acquires the same moments as an oscillator immersed in a Planck thermal bath superposed on the zero point.

Two structuring remarks. *(i)* In (19) the thermal part and the zero-point part are welded into a single analytic function, coth; Boyer reads this as a hint that Planck's law is a consequence of Relativity applied to zero-point radiation, rather than a primitive quantum fact. *(ii)* The temperature depends only on a: it is *universal*, independent of the detector's properties — a property which section 8 will connect to the possibility of letting horizons play a geometric role.

3.4 The electromagnetic case and the non-Planckian term

For the electromagnetic field, the parallel derivation (Boyer [5]) gives, for the spectral energy density seen by the accelerated observer,

$$\rho_a(\omega) = \frac{\hbar\,\omega^3}{\pi^2 c^3}\left(1 + \frac{a^2}{c^2\omega^2}\right)\left[\frac{1}{2} + \frac{1}{e^{\,2\pi\omega c/a}-1}\right], \tag{20}$$

which recovers (5) as $a \to 0$. The factor $\left(1 + a^2/c^2\omega^2\right)$ is a *non-Planckian* term, also present in the quantum computation for the spin-1 field in $3+1$ dimensions (see Takagi's review [11]): the thermal analogy is exact in the detailed-balance sense (16) — the temperature is indeed T_U — but the full energy content of the sampled bath retains a polynomial memory of a. This detail will matter for any theory, MiHsC foremost, that puts the *energy* of the Unruh bath to work and not merely its temperature.

4 The Rindler horizon: what becomes of entanglement? [Established]

4.1 Recap of the quantum version: the vacuum as a thermofield double state

The observer (8) is confined to the right Rindler wedge $R = \{x > c|t|\}$; the left wedge $L = \{x < -c|t|\}$ is causally inaccessible to him, the horizon being the cone $x = \pm ct$. In quantum theory, the Minkowski vacuum, decomposed on the eigenmodes of the boost (Rindler modes) of proper frequencies ω, takes the form of a *thermofield double* state (Fulling, Davies, Unruh [7, 8, 9], and for the algebraic structure Bisognano–Wichmann [10]):

$$|0_M\rangle \;\propto\; \bigotimes_{\omega} \sum_{n=0}^{\infty} e^{-\pi n\omega c/a}\, |n\rangle_L^{(\omega)} \otimes |n\rangle_R^{(\omega)}, \tag{21}$$

and the partial trace over L hands the observer in R a density matrix that is exactly thermal at T_U. In this reading, Unruh thermality *is* traced-out entanglement: the correlations $\langle 0|\hat{\varphi}_L\hat{\varphi}_R|0\rangle \neq 0$ straddle the horizon, and the thermal entropy of the accelerated observer is the entanglement entropy of the bipartition (L, R).

4.2 The classical version: one draw, two wedges

What does Boyer's ether say? The answer fits in one sentence: *a realization of the random field is a global object over all of Minkowski; the two Rindler wedges are two windows opened onto the same draw.* Trans-horizon correlations therefore exist trivially. For instance, for two symmetric points at zero Minkowski time, $x_R = (0, \rho_0, 0, 0)$ and $x_L = (0, -\rho_0, 0, 0)$ (natural units):

$$C(x_L, x_R) = \frac{1}{16\pi^2\rho_0^2} \;\neq\; 0, \tag{22}$$

a power-law decay, with no screening whatsoever: *the ether ignores the horizon.* The horizon is not a property of the field, it is a property of the observer's *trajectory*, that is, of his sampling protocol. First conclusion, answering half of the question: in the classical version, there is *no physical*

filtering of the wavelengths that would "overflow" the horizon. All wavelengths exist, everywhere.

4.3 Mode-by-mode trans-horizon correlations: the computation

Let us sharpen the fine structure. Decompose the field, in each wedge, on the Rindler modes of proper frequency ω (referred to the proper time of the fiducial observer of acceleration a), with random amplitudes $c_{R,\omega}$ and $c_{L,\omega}$. The link with the Minkowski ensemble goes through the Unruh combinations: setting

$$\tanh r_\omega = e^{-\pi\omega c/a}, \tag{23}$$

the combinations

$$d_{1,\omega} = \cosh r_\omega \, c_{R,\omega} + \sinh r_\omega \, \bar{c}_{L,\omega}, \qquad d_{2,\omega} = \cosh r_\omega \, c_{L,\omega} + \sinh r_\omega \, \bar{c}_{R,\omega}, \tag{24}$$

are genuine positive-frequency Minkowski modes. Boyer's ensemble is then characterized by: each $d_{i,\omega}$ carries the pure zero point and the d's are mutually uncorrelated,

$$\langle |d_{i,\omega}|^2 \rangle = \tfrac{1}{2}, \qquad \langle d_1 d_2 \rangle = \langle d_1 \bar{d}_2 \rangle = 0. \tag{25}$$

Inverting (24), $c_{R,\omega} = \cosh r_\omega \, d_{1,\omega} - \sinh r_\omega \, \bar{d}_{2,\omega}$, one obtains the second moments of the wedge modes:

$$\big\langle |c_{R,\omega}|^2 \big\rangle = \big\langle |c_{L,\omega}|^2 \big\rangle = \tfrac{1}{2} \cosh 2r_\omega = \tfrac{1}{2} \coth\!\Big(\frac{\pi\omega c}{a}\Big), \tag{26}$$

$$\big\langle c_{R,\omega} \, c_{L,\omega} \big\rangle = -\tfrac{1}{2} \sinh 2r_\omega = -\,\frac{1}{2\sinh(\pi\omega c/a)}. \tag{27}$$

Equation (26) recovers, mode by mode, the spectrum (19): the thermality seen from the right wedge is the *marginal* law of a joint Gaussian. Equation (27) is its hidden counterpart: the amplitudes in the two wedges are phase-correlated. The normalized correlation coefficient reads

$$\boxed{\gamma(\omega) = \frac{\big|\langle c_{R,\omega} c_{L,\omega} \rangle\big|}{\sqrt{\langle |c_{R,\omega}|^2 \rangle \, \langle |c_{L,\omega}|^2 \rangle}} = \frac{1}{\cosh(\pi\omega c/a)}} \tag{28}$$

with the two telling limits

$$\gamma(\omega) \underset{\omega\to 0}{\longrightarrow} 1, \qquad \gamma(\omega) \simeq 2\, e^{-\pi\omega c/a} \quad (\omega \gg a/c). \tag{29}$$

4.4 Answer to the question posed: marginalization, not filtering

We can now answer, at three levels, the question "filtering of wavelengths or suppression of correlations?".

First level: nothing is filtered. The classical field is global, all wavelengths persist and the correlations cross the horizon with the power law (22). The horizon cuts nothing.

Second level: entanglement is replaced by a classical statistical correlation, graded in frequency. What quantum theory describes as the entanglement of the thermofield double (21) becomes, in Boyer's ether, the Gaussian correlation (27) between mirror modes of the two wedges, with coefficient $\gamma(\omega) = 1/\cosh(\pi\omega c/a)$. The low-Rindler-frequency modes — those whose wavelength, measured in the accelerated frame, exceeds the distance to the horizon $d_H = c^2/a$ — are almost perfectly correlated from one wedge to the other ($\gamma \to 1$): they are the *common property* of both sides. The high-frequency modes are exponentially "private" ($\gamma \sim 2e^{-\pi\omega c/a}$). This is the exact form taken by the geometric filtering intuition: it is not the waves that are cut by the horizon, it is the *amplitude correlations* that are shared or private according to whether the wave "fits" in the wedge or not.

Third level: thermality is a marginalization. The Rindler observer only has access to the modes of his wedge; the marginal distribution of the joint Gaussian (26)–(27) is exactly the thermal ensemble at T_U superposed on the zero point. The quantum partial trace is replaced by the marginalization of a joint probability law — a mundane operation of probability theory, with neither collapse nor mystery.

Where, then, does the quantum/classical boundary lie? The second moments (26)–(27) are exactly those of the two-mode squeezed state (21): the Wigner function of the vacuum restricted to two wedges is a positive Gaussian, and Boyer's ether is precisely that Gaussian taken as a genuine probability. A remarkable operational fact follows: *no Gaussian measurement (quadratures, homodyning, quadratic energies) can distinguish the two descriptions.* The difference resides only in non-Gaussian measurements (photon counting, parity) and in Bell tests, which classical correlations cannot violate. In other words: across the entire experimental sector of the Unruh effect envisioned to date, the "classical correlations" reading and the "entanglement" reading are empirically degenerate. This is excellent news for speculative freedom: the terrain is genuinely open.

4.5 The cost of true censorship: the lesson of the Boulware state

A thought experiment pins down the ideas for what follows. Can one build a classical ensemble in which the trans-horizon correlations are *genuinely zero* — the analogue of the Fulling–Rindler vacuum (or, for a black hole, of the Boulware state)? Yes: it suffices to draw the amplitudes $c_{R,\omega}$ and $c_{L,\omega}$ independently with the Rindler zero point $\langle|c|^2\rangle = \frac{1}{2}$. But this ensemble, regular for the accelerated observer, is pathological from the Minkowski viewpoint: its two-point function differs from (3) by a term that *diverges on the horizon*, and the relative energy density grows like ϱ^{-4} as the proper distance ϱ to the horizon tends to zero (classical result on the quantum side: [12, 13]). The moral, which will serve as a structuring constraint for the next section: *hard-decorrelating the two sides of a horizon costs a divergent energy localized on the horizon.* Any theory that puts a horizon to work as a censor of radiation — MiHsC first among them — must therefore either assume an energetic horizon layer, or soften the censorship into a gradual suppression, of which (28) provides the natural prototype.

5 McCulloch's Quantized Inertia reread through Boyer's ether

5.1 Structured review [Review]

The MiHsC theory (*Modified inertia by a Hubble-scale Casimir effect*, also called *Quantised Inertia*) of McCulloch [16, 17, 18] rests on two postulates. *(P1)* The inertia of a body comes from the Unruh radiation it sees when it accelerates: a body accelerated to the right acquires a Rindler horizon at distance $d_H = c^2/|a|$ behind it; this radiation is depleted anisotropically (more strongly on the horizon side), and the resulting net radiation pressure opposes the acceleration — that is inertia. *(P2)* At very low acceleration, the Unruh wavelengths become cosmological and undergo in addition a Casimir effect at the Hubble scale: only the waves that "fit" within the cosmic horizon of diameter Θ are allowed, i.e. $\lambda_n = 2\Theta/n$.

Quantitatively, Wien's law applied to T_U gives the dominant wavelength of the Unruh spectrum,

$$\lambda_U \simeq \frac{2\pi c\, k_B b_{\mathrm{Wien}}}{\hbar\, a} \approx \frac{8c^2}{a}, \tag{30}$$

and the linear depletion of the spectrum by the cosmic horizon leads to the modified inertial mass

$$\boxed{m_i = m_g \left(1 - \frac{\lambda_U}{4\Theta}\right) = m_g \left(1 - \frac{2c^2}{|a|\,\Theta}\right).} \tag{31}$$

From this follows a minimum acceleration below which inertia collapses,

$$a_{\min} = \frac{2c^2}{\Theta} \;\approx\; 7 \times 10^{-10}\ \mathrm{m\,s^{-2}}, \tag{32}$$

of the same order as the observed cosmic acceleration and as the MOND scale a_0. In the deep regime ($a \ll a_{\min}$ being impossible, the system settles at $a \sim a_{\min}$), the gravitational equilibrium of a galaxy gives a flat velocity

$$v^4 = \frac{2GMc^2}{\Theta}, \tag{33}$$

i.e. the baryonic Tully–Fisher relation without dark matter, with a_0 *predicted* rather than fitted — the theory's flagship argument, applied by its author to rotation curves, flyby anomalies and cluster dynamics.

Genealogy. MiHsC is the direct heir of the SED programme for inertia: Haisch, Rueda and Puthoff [14, 15] had already proposed that the inertial force be the reaction of the classical zero-point field on an accelerated scatterer (a zero-point Lorentz-type force, proportional to a). The difference is the asymmetry mechanism: for HRP, a coupling and a cutoff; for McCulloch, *censorship by horizons* — exactly the object that section 4 has just analysed. MiHsC thus sits, structurally, at the intersection of Boyer's ether and horizon thermodynamics.

5.2 Correlational reformulation [Speculation]

Section 4 showed that the rigorous classical version of horizon censorship is not a guillotine on wavelengths but the grading of correlations (28). Let us therefore propose to refound (P1) on this basis.

Ansatz A (private energy). Postulate that only the fraction of the Unruh bath *uncorrelated* with the beyond-horizon region can exert a net reaction on the accelerated body — the correlated fraction, being "co-owned" by inaccessible degrees of freedom, can carry no net momentum exchange. The effective spectral density becomes

$$u_{\mathrm{priv}}(\omega) = u_a(\omega)\,\big[1 - \gamma^2(\omega)\big] = u_a(\omega)\,\tanh^2\!\Big(\frac{\pi\omega c}{a}\Big). \tag{34}$$

The suppression is soft (quadratic in ω at low frequency), where MiHsC cuts sharply: (34) is the "softened Boulware" version compatible with the energetic lesson of section 4.5.

Ansatz B (the cosmic horizon as an infrared bound). The cosmic horizon imposes on the Rindler spectrum a minimum proper frequency $\omega_{\min} \sim 2\pi c/\Theta$ (the gravest Rindler wave that fits within the observable universe), whether the cutoff is hard (Casimir–Dirichlet, MiHsC spirit) or soft (correlational, the spirit of Ansatz A applied to the cosmic horizon).

Proposition 2 (Robust scaling law) *Any infrared depletion of the Unruh spectrum below a frequency $\omega_{\min} \sim c/\Theta$, whatever its precise form, produces a relative energy deficit of the bath*

$$\delta(a) \;=\; \xi\,\frac{c^2}{a\,\Theta} \;+\; O\Big(\big(c^2/a\Theta\big)^2\Big), \tag{35}$$

where $\xi = O(1\text{–}10)$ depends on the filter but not on the scale. If inertia is proportional to the available energy of the bath, then $m_i = m_g\,[1-\xi\,c^2/(a\Theta)]$: the MiHsC form (31) is universal *in the regime $a\Theta \gg c^2$, only the coefficient ξ being a model choice (MiHsC: $\xi = 2$).*

Sketch of proof (natural units). The thermal part of the worldline spectrum (19) is $S_{\rm th}(\omega) = \frac{\omega}{2\pi}(e^{2\pi\omega/a}-1)^{-1}$. At low frequency ($\omega \ll a$), $S_{\rm th} \to a/4\pi^2$: the infrared weight is *flat*, so the energy removed by any suppression below $\omega_{\min}$ is $\sim (a/4\pi^2)\,\omega_{\min}$ up to the filter's form factor. The total thermal energy is $\int_0^\infty S_{\rm th}\,\mathrm{d}\omega = a^2/48\pi$. The ratio is $\propto \omega_{\min}/a \propto c/(a\Theta)$, i.e. (35) after restoring units ($c^2/a\Theta$ is the dimensionless ratio $\hbar\omega_{\min}/k_B T_U$ up to 2π). □

This proposition is, to our knowledge, the most useful point that the correlational analysis brings to MiHsC: *the phenomenology of the theory does not depend on its most contested hypothesis* (the hard Casimir-type filtering). A soft suppression, energetically clean in the sense of section 4.5, gives the same law $1-\xi c^2/(a\Theta)$, the same minimum acceleration $a_{\min} = \xi c^2/\Theta$ and the same Tully–Fisher relation $v^4 = \xi GMc^2/\Theta$, up to the value of ξ.

The covariant Deser–Levin floor. General Relativity moreover offers a "native" mechanism for the scale $a_{\min}$, without any filtering: in a de Sitter spacetime of Hubble constant H, a detector of proper acceleration a sees the temperature (Deser–Levin [19], generalizing Gibbons–Hawking [21])

$$T_{\rm eff} = \frac{\hbar}{2\pi c\,k_B}\sqrt{a^2 + c^2H^2}. \tag{36}$$

The Unruh bath never switches off: it saturates at the Gibbons–Hawking floor as $a \to 0$, and the crossover scale is precisely $a_c = cH = 2c^2/\Theta$ for Θ the Hubble diameter — the MiHsC scale (32), recovered by pure geometry. A covariant variant of quantized inertia then reads: $m_i = m_g\,f\big(a/\sqrt{a^2+c^2H^2}\big)$ with $f(1) = 1$, interpolating smoothly instead of vanishing. The two branches

($f \to 0$ MiHsC-style, or saturation in the style of (36)) are empirically discriminable, for instance on wide binaries and tidal dwarves, which makes this bifurcation a genuine observational programme rather than a quarrel of principles. Section 6 takes up this role of the second horizon systematically and constructs the extinction mechanism.

Structuring constraints (not verdicts). Three worksites remain open and fertile. *(a)* Energy bookkeeping: if the horizon depletes the bath, who pays? The natural answer within the framework of section 4.5 is a horizon surface tension, to be computed. *(b)* Covariance: the Rindler horizon is observer-dependent; the version (36) shows the way toward a geometric formulation. *(c)* Equivalence principle: since (31) depends only on a, the modification is universal in composition — the weak equivalence principle is preserved, which is remarkable for a theory of emergent inertia; it is its strong version (locality) that is at stake, through the dependence on Θ.

6 The cosmological horizon: the second horizon and the extinction of inertia

The previous section treated the cosmological horizon as a mere infrared bound. We now integrate it as a full-fledged object of the study, on the same footing as the Rindler horizon — and we show that it is precisely the *pair* of horizons that explains the disappearance of inertia at low acceleration.

6.1 Two horizons for a single observer [Established]

A body of proper acceleration a, in a universe undergoing accelerated expansion with Hubble constant H (de Sitter background), is framed by two horizons of different natures but symmetric roles. Behind it, the Rindler horizon, at proper distance

$$d_H = \frac{c^2}{a} \ : \tag{37}$$

a *personal*, directional horizon, which exists only as long as the acceleration lasts. All around it, the cosmological horizon, at distance

$$R_H = \frac{c}{H} = \frac{\Theta}{2} \ : \tag{38}$$

a *shared*, isotropic horizon, independent of the state of motion. The comparison of the two distances defines the hinge scale of everything that follows:

$$d_H < R_H \quad \Longleftrightarrow \quad a > a_c \equiv cH = \frac{2c^2}{\Theta} \approx 6.6 \times 10^{-10}\ \mathrm{m\,s^{-2}}. \tag{39}$$

At high acceleration, the personal horizon is close and the cosmic horizon far away: the physics of section 4 applies as is. As a decreases, d_H recedes;

at $a = a_c$, the Rindler horizon *reaches* the Hubble horizon, and below that the very notion of a personal rear horizon dissolves into the isotropic cosmic horizon. The entire low-acceleration phenomenology is the translation of this geometric merger.

6.2 The two-horizon sampling theorem [Established]

The central result is that *all* the derivations of sections 3 and 4 carry over to the two-horizon case by a single substitution. De Sitter space embeds as a hyperboloid in a five-dimensional Minkowski space, and a uniformly accelerated worldline of de Sitter lifts there to a Rindler hyperbola of effective acceleration (GEMS approach, Narnhofer–Peter–Thirring and Deser–Levin [20, 19])

$$\boxed{\kappa = \sqrt{a^2 + c^2 H^2}} \tag{40}$$

For the massless conformally coupled scalar field in the Bunch–Davies state, the Wightman function sampled along this trajectory is exactly of the form (12) with $a \to \kappa$ (natural units):

$$W(\Delta\tau) = -\frac{\kappa^2}{16\pi^2} \frac{1}{\sinh^2\!\left[\frac{\kappa}{2}(\Delta\tau - i\varepsilon)\right]}, \tag{41}$$

whence, by the lemma of section 3, the spectrum $S_\kappa(\omega) = \frac{\omega}{4\pi}\coth(\pi\omega/\kappa)$, the Deser–Levin temperature (36), and the trans-horizon correlation structure of section 4 with

$$\gamma_\kappa(\omega) = \frac{1}{\cosh(\pi\omega c/\kappa)}. \tag{42}$$

Two consequences deserve to be set apart. *(i)* For the comoving observer ($a = 0$), $\kappa = cH$: the cosmological horizon reproduces identically the correlational structure of the Rindler horizon. The answer of section 4 therefore extends word for word to the cosmos: nothing is filtered by the Hubble horizon, the field is globally correlated across it, the Gibbons–Hawking bath is the marginal law of a joint Gaussian, and the super-horizon modes are the common property of inside and outside ($\gamma \to 1$). *(ii)* The acceleration signal *rides on an incompressible floor*: the part of the bath attributable to a is the difference $\kappa - cH$, never the whole bath.

6.3 The rigorous seed of the extinction [Established]

Before any speculation, an elementary fact follows from (40):

$$\frac{\partial\kappa}{\partial a} = \frac{a}{\kappa} \xrightarrow[a \ll a_c]{} \frac{a}{cH} \to 0, \qquad \kappa - cH \simeq \frac{a^2}{2cH} \quad (a \ll a_c). \tag{43}$$

Below a_c, the bath's dependence on the acceleration becomes *quadratic*: the sampled bath no longer carries any information at first order in a. Now an

inertial force is, by definition, a reaction *linear* in a. No functional of the bath — whatever theory of emergent inertia is adopted — can manufacture a linear response out of a quadratic source: the bath-borne inertial response must therefore degrade below a_c. This much is theorem-level. What remains model-dependent is the shape of the degradation: saturation of m_i toward a residual value, or complete extinction. The next subsection builds the mechanism of the extinction.

6.4 The correlational extinction mechanism [Speculation]

Take up Ansatz A of section 5.2 — only the *private* energy (uncorrelated beyond a horizon) can exert a net reaction — and make it directional. For a mode of proper frequency ω propagating toward a direction in which the effective horizon lies at proper distance d, set, generalizing (28),

$$\gamma_d(\omega) = \frac{1}{\cosh(\pi\omega d/c)}, \tag{44}$$

which recovers (28) for the Rindler horizon ($d = c^2/a$). Toward the *rear* of the accelerated body, the effective horizon is the Rindler horizon, $d = d_H = c^2/a$; toward the *front*, there is only the cosmic horizon, $d = R_H = \Theta/2$. Define the *shared* fraction of the thermal bath in a direction with horizon at distance d:

$$D(d) = \frac{\displaystyle\int_0^\infty \mathrm{d}\omega\, S_{\mathrm{th}}(\omega)\,\gamma_d^2(\omega)}{\displaystyle\int_0^\infty \mathrm{d}\omega\, S_{\mathrm{th}}(\omega)}, \qquad S_{\mathrm{th}}(\omega) = \frac{\omega}{2\pi}\,\frac{1}{e^{2\pi\omega/\kappa}-1}. \tag{45}$$

Two evaluations (natural units, regime $a \gg a_c$ so $\kappa \simeq a$):

Rear. With $d = 1/a$, the integrand depends only on $x = \omega/a$: the fraction shared with the beyond of the Rindler horizon is a pure number, independent of the scale,

$$D_{\mathrm{rear}} = \kappa_0 = \frac{\int_0^\infty x\,(e^{2\pi x}-1)^{-1}\cosh^{-2}(\pi x)\,\mathrm{d}x}{\int_0^\infty x\,(e^{2\pi x}-1)^{-1}\,\mathrm{d}x} \approx 0.657. \tag{46}$$

This is the self-similarity of the Rindler problem: about two thirds of the Unruh bath is, at any acceleration, co-owned with the beyond of the personal horizon.

Front. With $d = R_H \gg c/\kappa$, only the infrared band $\omega \lesssim c/R_H$ is shared; since the infrared weight of the spectrum is flat ($S_{\mathrm{th}} \to \kappa/4\pi^2$) and $\int_0^\infty \cosh^{-2}(\pi\omega R_H/c)\,\mathrm{d}\omega = c/\pi R_H$,

$$D_{\mathrm{front}} = \frac{(\kappa/4\pi^2)(c/\pi R_H)}{\kappa^2/48\pi} = \frac{12\,c}{\pi^2\kappa R_H} = \frac{24}{\pi^2}\,\frac{c^2}{a\,\Theta} \approx 2.43\,\frac{c^2}{a\Theta}, \tag{47}$$

where units have been restored. Note that the coefficient $24/\pi^2 \approx 2.4$, arising from the soft correlational filter, lands remarkably close to the coefficient 2 that McCulloch obtains through his hard Casimir-type filtering (31): a new, this time quantitative, illustration of the robust scaling law (35).

The asymmetry and inertia. The inertial reaction is carried by the front–rear asymmetry of the private energy:

$$\mathcal{A}(a) = \big[1 - D_{\text{front}}(a)\big] - \big[1 - D_{\text{rear}}\big] = \kappa_0 - \frac{24}{\pi^2}\,\frac{c^2}{a\Theta}. \tag{48}$$

More energy "capable of pushing" arrives from the front than from the rear; the imbalance opposes the acceleration, and inertia is proportional to it. Normalizing $m_i(a \to \infty) = m_g$:

$$\boxed{m_i(a) = m_g\,\frac{\mathcal{A}(a)}{\mathcal{A}(\infty)} = m_g\left[1 - \xi\,\frac{c^2}{a\,\Theta}\right], \qquad \xi = \frac{24}{\pi^2\kappa_0} \approx 3.7,} \tag{49}$$

i.e. the MiHsC form (31) with a coefficient of the same order, and complete extinction at

$$a_{\min} = \xi\,\frac{c^2}{\Theta} \approx 1.2 \times 10^{-9}\ \mathrm{m\,s^{-2}}. \tag{50}$$

Why inertia disappears: three equivalent readings. *Geometric*: as $a \to a_{\min} \sim a_c$, the Rindler horizon recedes all the way to the Hubble horizon (39); the front and the rear of the body then see horizons at the same distance, and nothing, in the horizon structure, distinguishes the direction of the acceleration any more. *Correlational*: the shared fractions equalize, $D_{\text{front}} \to D_{\text{rear}}$, the private-energy asymmetry (48) vanishes — the bath becomes statistically isotropic again as far as its capacity to push is concerned, and the reaction that constituted inertia no longer has a support. *Thermodynamic*: the personal Unruh temperature sinks below the Gibbons–Hawking floor (36); the signal in a becomes quadratic (43) and drowns in the isotropic cosmic bath, which does not know in which direction the body is accelerating. The three readings say the same thing: *inertia is the statistical trace of the horizon anisotropy, and it dies when the personal horizon dissolves into the shared one.*

Consequences and energetic cleanliness. Below $a_{\min}$, no stationary accelerated state is self-consistent: a bound system settles at $a \simeq a_{\min}$, which recovers the flatness of rotation curves and the Tully–Fisher relation (33), now derived from the correlational mechanism rather than from a filtering postulate. The filter γ_d^2 being smooth, the decorrelation is soft in the sense of section 4.5: no divergent energy layer is required on either horizon — the cosmic horizon even supplies for free, through the Bunch–Davies state, exactly the grading (42) that the Ansatz postulates. There remain

the honest limits of the exercise: a toy scalar field, a heuristic directional ansatz (44) (the assignment of an effective horizon per direction is an eikonal approximation), a coefficient ξ not finely predicted; but the proposition of section 5.2 guarantees that the scaling law, the existence of $a_{\min}$ and its order of magnitude survive all these refinements.

7 Extending the covariant ether to General Relativity

7.1 From global to local invariance: the Hadamard condition [Established]

In General Relativity, Lorentz invariance is no longer global but local: the equivalence principle demands that, in every locally inertial frame, short-distance physics be that of Minkowski. Translated for a Gaussian random field on a manifold $(\mathcal{M}, g)$, this fixes the *short-distance form* of the correlation function: it is precisely the Hadamard condition of field theory in curved spacetime [22]. For the massless scalar field,

$$C(x,x') = \frac{\hbar}{8\pi^2}\left[\frac{2\,\Delta^{1/2}(x,x')}{\sigma(x,x')} + v(x,x')\,\ln\frac{|\sigma(x,x')|}{\ell^2} + w(x,x')\right], \qquad (51)$$

where $\sigma(x,x')$ is Synge's world function (half the squared geodesic distance), $\Delta^{1/2}$ the van Vleck–Morette determinant, and v, w regular biscalars whose expansions are fixed by the geometry (v) and by the state (w). The singular part is *universal and purely geometric*: in the tangent space it reduces exactly to (3). In other words:

Proposition 3 *The covariant generalization of Boyer's ω^3 spectrum is the Hadamard condition: a "Boyer field on a manifold" is a Gaussian random field whose correlation has the form (51). The ω^3 law is the fingerprint, in each tangent space, of the Hadamard singularity.*

7.2 The choice of state as statistical boundary conditions [Established]

In curved spacetime there is no unique vacuum: the regular part w of (51) remains free, and its choice corresponds, in the stochastic language, to the choice of the *boundary conditions of the random ensemble*. The dictionary is instructive for Schwarzschild: the Boulware state (stationary, without trans-horizon correlations, singular on the horizon — the "censored" field of section 4.5); the Hartle–Hawking state (correlated across the horizon, regular, thermal at $T_H = \hbar c^3/8\pi G M k_B$); the Unruh state (regular on the future horizon, outgoing flux — evaporation). In de Sitter, the Bunch–Davies state

gives the Gibbons–Hawking floor already encountered in (36). The "classical Hawking" programme consists in redoing, for the Hadamard correlation near a black-hole horizon, what section 3 did for Rindler: the stationary sampling of the regular correlation delivers the thermal factor at T_H by the same KMS mechanism (16), thermality remaining a sampling theorem and not a pair-creation process.

7.3 Back-reaction: the Einstein–Langevin equation [Established]

How does the random field act back on the geometry? The formalism exists and goes by the name of stochastic gravity (Hu–Verdaguer [23]): beyond the mean semiclassical equation, the metric responds to the *fluctuations* of the stress-energy tensor through the Einstein–Langevin equation

$$G_{\mu\nu}[g+h] = \frac{8\pi G}{c^4}\Big(\langle T_{\mu\nu}\rangle_{\mathrm{ren}} + \xi_{\mu\nu}\Big), \qquad \langle \xi_{\mu\nu}(x)\,\xi_{\rho\sigma}(y)\rangle = N_{\mu\nu\rho\sigma}(x,y), \quad (52)$$

where the noise kernel N is the *symmetric part* of the connected correlations of $T_{\mu\nu}$. Note the striking structural convergence: stochastic gravity is, by construction, the theory of the symmetric part — exactly the sector that Boyer's ether reproduces (7). The extension of stochastic electrodynamics to General Relativity therefore already exists under another name: it is a "gravitational SED".

The major structuring constraint: the cosmological constant. The ω^3 spectrum integrated up to the Planck scale gives

$$\rho_{\mathrm{ZPF}} = \int_0^{\omega_P} \frac{\hbar\,\omega^3}{2\pi^2 c^3}\,\mathrm{d}\omega = \frac{\hbar\,\omega_P^4}{8\pi^2 c^3} \sim 10^{112}\ \mathrm{J\,m^{-3}}, \quad (53)$$

against $\rho_\Lambda^{\mathrm{obs}} \approx 5\times 10^{-10}\ \mathrm{J\,m^{-3}}$: about 120 orders of magnitude. Any extension of the ether to GR must say why the homogeneous part of the zero point does not gravitate (or gravitates with $p=-\rho$ and cancels), only its *correlation inhomogeneities* sourcing (52). This is a problem shared with quantum field theory — the ether is neither better nor worse off — but it becomes central as soon as one wants the field to *engender* the geometry (section 8).

7.4 Proposed axiomatics and inertia in curved spacetime [Speculation]

We propose to define a *Boyer field on a manifold* by the following axioms.

A1. Real Gaussian random field φ on $(\mathcal{M}, g)$, a solution (realization by realization) of $\Box_g \varphi = 0$.

A2. Correlation of Hadamard form (51) (local Lorentz invariance, tangent ω^3 spectrum).

A3. If $(\mathcal{M}, g)$ admits a timelike Killing field, the statistics are stationary along it.

A4. Horizons (Killing or cosmological) carry the statistical boundary conditions of the ensemble, with the energetic cost of section 4.5 for any decorrelation.

Under A1–A4, the Unruh, Hawking and Gibbons–Hawking temperatures become *sampling theorems* of the same type as (19). Quantized inertia is formulated covariantly there: the inertial mass of a scatterer of cross-section $\varsigma(\omega)$ along a worldline is a functional of the local noise kernel, schematically $m_i \propto c^{-2} \int \mathrm{d}\omega\, \varsigma(\omega)\, u_{\mathrm{priv}}(\omega)$ with the private energy (34) evaluated for the effective horizons of the worldline, and the floor (36) guaranteed by A4 in the presence of a cosmological horizon.

8 From radiation to the metric [Established, then Speculation]

8.1 The Minkowski metric is reconstructed from the correlations [Established]

Let us pose the question in its weak sense: does the correlation function (3) alone suffice to recover $\eta_{\mu\nu}$? The answer is yes, and the formula is explicit. Near coincidence, the Hadamard form (51) gives $C \simeq \hbar/(4\pi^2 \cdot 2\sigma)$, hence

$$2\,\sigma(x,x') = \frac{\hbar}{4\pi^2\, C(x,x')} + \cdots , \tag{54}$$

and since $\sigma(x,x') = \frac{1}{2} g_{\mu\nu}(x) \Delta x^\mu \Delta x^\nu + O(\Delta x^3)$,

$$\boxed{\, g_{\mu\nu}(x) = -\,\frac{1}{2} \lim_{x' \to x} \frac{\partial^2}{\partial x^\mu\, \partial x'^\nu} \left[\frac{\hbar}{4\pi^2\, C(x,x')} \right] . \,} \tag{55}$$

Flat-space check: $C = \hbar/[4\pi^2\, \eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta]$ (spacelike side, principal value), the bracket equals $\eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta$, and $-\frac{1}{2}\, \partial_\mu \partial_{\nu'} [\eta_{\alpha\beta} \Delta x^\alpha \Delta x^\beta] = \eta_{\mu\nu}$. The Minkowski metric is therefore *entirely encoded* in Boyer's radiation, and recoverable by a double differentiation of the inverse of the correlation.

A second, more conceptual route deserves mention: the *singular support* of C — the pairs of points where the correlation diverges — is exactly the set of pairs connected by a light ray. It therefore defines the cone field, that is, the causal structure; and the theorems of Hawking–King–McCarthy and Malament [27, 28] guarantee that the causal structure determines the

metric up to a conformal factor. The remaining conformal factor is fixed by the universal normalization $\hbar/4\pi^2$ of the Hadamard singularity (the "volume" that the zero point attributes to each spectral cell). Causal structure = locus of infinite correlations; scale = intensity of the zero point: the geometry can be read off the statistics.

8.2 Engendering geometry: conditions and programmes [Speculation, backed by reviews]

Now the strong sense: under what conditions could a field *engender* a geometry — that is, the metric no longer being a pre-existing stage but a *summary* of the correlations?

The self-consistent fixed point.

Definition 1 (Self-consistent pair) *A pair (C, g) is self-consistent if (i) $g = g[C]$ through the reconstruction (55), and (ii) C is the symmetric Hadamard correlation of a stationary Gaussian field solving $\Box_g \varphi = 0$.*

Proposition 4 *The pair ($\eta_{\mu\nu}$, ω^3 spectrum) is self-consistent: Minkowski is a fixed point of the bootstrap.*

Conjecture 1 (Rigidity) *Under homogeneity, isotropy and statistical scale invariance, the only fixed points are Minkowski and the (anti-)de Sitter family equipped with its Bunch–Davies-type states, parametrized by a constant Λ.*

The bootstrap says which geometries are *compatible* with their own noise; it does not yet say why the geometry *obeys Einstein*. Two existing programmes supply that link, and the ether slots into both naturally.

Sakharov's induced gravity. Sakharov [24, 25] observes that the effective action obtained by integrating the fluctuations of a field on a background g automatically contains, through the heat-kernel expansion, the terms $\sqrt{-g}$ and $\sqrt{-g}\,R$:

$$\Gamma_{\rm ind}[g] \;\supset\; c_0 \Lambda_c^4 \int \sqrt{-g} \;+\; c_1 \Lambda_c^2 \int \sqrt{-g}\, R, \qquad \frac{1}{G_{\rm ind}} \sim \frac{N\,\Lambda_c^2}{12\pi} \quad (\hbar = c = 1), \tag{56}$$

with Λ_c the cutoff and N the number of fields. The rigidity of spacetime (Newton's constant) would be the *elasticity of the zero point*: the metric resists deformation because deforming g deforms the Hadamard correlations and costs zero-point energy. In this reading, Boyer's ether does not inhabit spacetime: its stiffness *is* spacetime. The price, inherited from the previous subsection, is the Λ_c^4 term — the cosmological-constant problem in its crudest form.

Jacobson's equation of state. Jacobson [26] derives the Einstein equation as a thermodynamic equation of state: imposing, on every local Rindler horizon, the Clausius relation $\delta Q = T\,\delta S$ with T the local Unruh temperature and $S = \eta A$ proportional to the area, the Raychaudhuri equation turns this requirement into $G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G\, T_{\mu\nu}$. Reread his three ingredients in the ether: the temperature T_U is supplied *classically* by the sampling theorem (19); the heat flux δQ is the energy flux of the stochastic field through the horizon; and the area entropy, finally, can be motivated classically: the mutual information of a Gaussian field between the two sides of a surface diverges as A/ϵ^2 — an area law, exactly like entanglement entropy, the cutoff ϵ playing the role that Sakharov assigns to Λ_c^{-1} in fixing G. The two programmes close upon each other.

Conditions for a field to engender a geometry. In synthesis, we propose that a statistical field engenders a geometry if it satisfies:

C1. a *universal* short-distance correlation singularity (Hadamard type), which defines cones and metric via (55);

C2. hyperbolic self-consistency: the characteristics of its own dynamics coincide with the reconstructed cones (the field propagates on the geometry it draws);

C3. an area law of trans-boundary information with a universal coefficient, which supplies Jacobson's horizon entropy;

C4. an acceleration–temperature relation of KMS type (16), which supplies the T of Clausius.

Boyer's ether satisfies C1, C2 and C4 *exactly*, and C3 up to a regularization. The answer to the initial question is therefore: *yes*, the Minkowski metric can be recovered from Boyer's radiation; and the correlations required for a field to engender a geometry are, in essence, those this radiation already possesses.

The sharpest open question: does geometry need Bell? The quantum programme "entanglement = connectivity of spacetime" (Van Raamsdonk [29], ER=EPR [30]) relies on a property that classical correlations lack: *monogamy*. Entanglement is exclusive, classical correlation is polygamous. If geometric connectivity is counted by correlations, a classical ether paradoxically risks engendering a spacetime that is *too* connected — every region correlated with all the others with no limiting budget. Two speculative ways out: either effective monogamy is restored at the level of Gaussian second moments (Cauchy–Schwarz-type inequalities on γ playing the role of a budget), or Bell violation really is the certificate that geometry needs, and the

classical ether engenders the cones and the thermodynamics (C1–C4) but not the fine connectivity. Settling this point — for instance by seeking a Jacobson-type derivation that *fails* precisely when γ exceeds the quantum monogamy bounds — seems to us the deepest problem bequeathed by this study.

9 Conclusion and research programme

Let us gather the thread. Boyer's field is the unique background noise compatible with Special Relativity; sampled along a hyperbola, it becomes an Unruh thermal bath without quanta (19). At the Rindler horizon, nothing is filtered: entanglement is replaced by the graded classical correlation $\gamma(\omega) = 1/\cosh(\pi\omega c/a)$, and thermality by a marginalization — the complete answer to the central question of this document. This structure illuminates MiHsC: its law $1 - 2c^2/(a\Theta)$ is robust, independent of hard filtering, and its scale $a_{\min}$ has a covariant anchor in the Deser–Levin floor. The cosmological horizon, integrated as a second horizon (section 6), supplies the mechanism of the disappearance of inertia: the bath fractions shared toward the front (Hubble horizon) and toward the rear (Rindler horizon) equalize when $d_H = c^2/a$ reaches R_H, the private-energy asymmetry that constituted the inertial reaction vanishes, and $m_i \to 0$ at $a_{\min} = \xi c^2/\Theta$ with $\xi = 24/\pi^2\kappa_0 \approx 3.7$. The extension to General Relativity is marked out: the Hadamard condition for the spectrum, horizon boundary conditions for the state, the Einstein–Langevin equation for the back-reaction. Finally, the Minkowski metric is explicitly reconstructed from the correlations (55), and the conditions C1–C4 sketch what would be needed to pass from "recovering" to "engendering".

The research programme that emerges has four strands, in decreasing order of maturity: compute the horizon tension associated with a soft decorrelation (34) and check the energy budget of quantized inertia; carry out the full "classical Hawking" computation in Schwarzschild under axioms A1–A4; establish or refute the rigidity conjecture of the (C, g) bootstrap; and confront the monogamy question, that is, determine whether geometry demands correlations that only the commutator can supply. On each of these strands, the empirical degeneracy between the classical and quantum readings of the Gaussian sector — demonstrated in section 4 — guarantees that the speculation remains anchored in computations that both camps can verify.

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